# Novel Artificial Frequency Mapping Techniques for Multi-tone Simulation of Mixers

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***Abstract* — This paper describes two new artificial frequency mapping techniques suitable for the simulation of RF/Microwave mixers when excited by a multi-tone signal. Due to the gained computational efficiency, it is for the first time possible to simultaneously analyze small and large signal behavior of nonlinear mixers subject to real multi-tone signals. Besides the theoretical study, illustrative simulation results will also be given.**

## I. INTRODUCTION

The RF/Microwave Mixer is a fundamental block in nowadays communication systems. Because of its nonlinear nature, it generates nonlinear distortion that will degrade SNR needed for good communication quality.

Traditionally, this nonlinear block was simulated and measured using a two-tone input signal. Nowadays, with the recent growth in the wireless industry, new and more complex forms of spectra have to be simulated. Time-varying Volterra series has already been applied to multi-tone problems in mixers. But, because of its rapidly increasing complexity for orders higher then three, only small nonlinear problems can be analyzed [1].

Since the local oscillator, LO, and RF input signals can normally be assumed as uncorrelated, the alternative method presented in this paper uses the well-known harmonic balance, HB, coupled with an artificial frequency mapping technique, AFM, specially developed for this type of simulations.

Artificial frequency mappings can be applied to either diamond or box truncation spectra [2]. Since both of these truncation schemes have advantages and shortcomings, we will begin by discussing their main problems. Then, new ways of circumventing those difficulties will be given.

We begin by noting the improved simulation efficiency obtained with diamond truncation in non-frequency converters subject to equally spaced multi-tone excitations ($\omega_{RF} = \omega_{RF_0} + k\Delta\omega$, $k \in Z$), as compared to the one provided by box truncation [3]. This efficiency gain is due to the fact that diamond truncation mimics the natural way of generating nonlinear mixing products in band limited systems, thus saving frequency points of negligible amplitude.

Conversely, box truncation has shown to be more appropriate for cases where clear distinct orders are needed for each of the excitation tones. This is the case of small-signal mixers, where the diamond truncation looses effectiveness since it considers the maximum mixing order of the RF input signal, equal to the maximum mixing order needed for the much stronger local oscillator pump. In fact, in a typical non-saturating mixer, the best choice consists in using an appropriate mixing order for the RF, and a reasonably larger one for the LO. Nevertheless, the multi-tone RF signal nature determines that even if an asymmetric box truncation for the LO and RF were considered, the number of generated RF mixing terms would be much larger than the minimum needed to correctly represent it.

These facts ask for an optimized index vector generation, based on a mixed truncation scheme: box truncation for the LO (large-signal) and diamond truncation for the RF (small-signal). That constitutes the first AFM conceived to take profit of the specific characteristics of multi-tone mixer simulation.

If the RF and LO signals are both large, that is, if the RF amplitude is comparable to the pump, driving the mixer close to saturation, there is no point in treating them differently. In that case, a more efficient AFM, completely based on the diamond truncation scheme, is preferred.

The discussion of these novel two AFMs conceived for equally spaced multi-tone mixer simulation is the main objective of this paper. So, their underlining ideas will be first presented, followed by a correspondent performance comparison. Finally, an illustrative application is addressed where a real RF mixer will be simulated using those proposed mapped spectra.

## II. PROPOSED ARTIFICIAL FREQUENCY MAPPING TECHNIQUES

As anticipated in the Introduction, we begin by proposing an AFM based on a mixed truncation scheme:

diamond truncation for the RF equally spaced multi-tone signal, and box truncation for the LO. The algorithm necessary for its mapped frequency index generation is very simple, and can be divided into two steps.

The first step consists in applying the AFM for diamond truncation spectra previously developed for amplifiers [3] to our equally spaced multi-tone RF input. This mapped spectrum, composed of a certain number of adjacent clustered mixing products, is shown in Fig. 1.

In the second step, this diamond truncated mapped spectrum is considered as a new composite tone to be mixed with the LO. The resulting mixing products are then box truncated, Fig. 2, and another AFM considered. The obtained spectrum corresponds to the dense and periodic frequency index vector used for simulating the mixer.

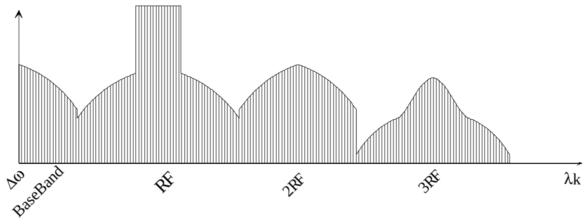


Fig.1 Output diamond truncated RF mapped spectrum.

Expression (1) presents the total number of resulting clusters considered in this case.

$$N \approx \left( \frac{O_{RF}+1}{2} O_{RF} + \frac{O_{RF}}{2}\left(O_{RF}-1\right) \right)\left(1+2O_{LO}\right) \qquad (1)$$

where $O_{RF}$ and $O_{LO}$ are the RF nonlinear order and the LO nonlinear order considered, respectively.

If the LO and the RF are comparable in magnitude, they should be treated uniformly. The traditional diamond truncation constitutes, thus, a better choice. In this case, the alternative AFM is based on a diamond truncation scheme specially designed for uniformly discretized spectra mixed with another uncorrelated LO tone ($\omega_{RF} = \omega_{RF_0} + k_1\Delta\omega$, but $\omega_{LO} \neq \omega_{RF_0} + k_2\Delta\omega$ , $k_1,k_2 \in$ Z). Fig. 3 presents the spectrum to be mapped.

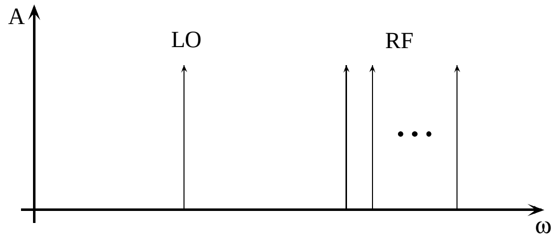


Fig. 3 Real mixer input spectrum.

The algorithm now proposed for the generation of the mapped spectrum, consists in three different steps.

The first step considers that the input spectrum is a two-tone signal. One of these imaginary tones is the local oscillator, while the other is located at the center of the RF signal. This generation gives rise to the mixing presented on Tab. 1, for the example of a third order nonlinearity. Each of these mixing terms creates a clustered spectrum.

With the knowledge of how much RF terms are produced in each cluster, the second step consists in generating the corresponding spectral regrowth, using the formulas already developed in [4].

Because the direct application of the general rules of diamond truncation AFM would not generate a periodic mapped spectrum, third step enforces that periodicity by inserting a certain number of zeros between spectrum clusters. This number of zeros can be determined by, first considering the LO and RF as a two-tone signal, and (as before) simply ignoring the zeros present between each of these clusters. Then, the various numbers of zeros between the output RF (spectral regrowth included) and the LO are determined for each cluster. The number of zeros required for guaranteeing a harmonically related mapped spectrum is equal to the minimum of those. This way, it is possible to generate a new artificially mapped spectrum that is periodic and compact. Although not ideally optimum, the resulting final spectrum is much more efficient to handle than the original one. Fig. 4 depicts the resulting mapped spectrum, while Tab. 2 presents the number of frequency positions required for a large number of input tones.

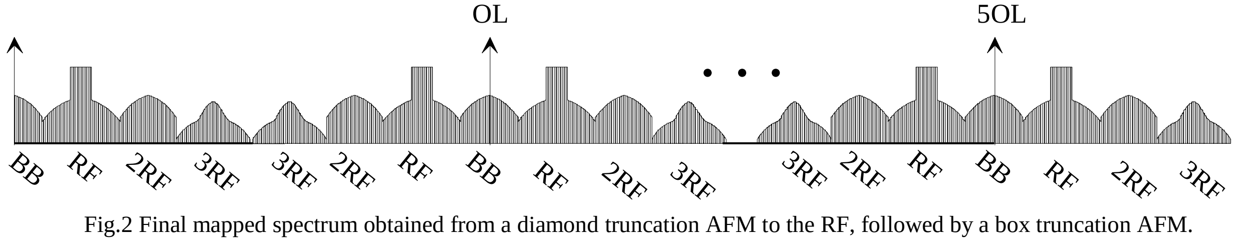


Fig.2 Final mapped spectrum obtained from a diamond truncation AFM to the RF, followed by a box truncation AFM.

Tab.1– Two tone cluster generation for a third order nonlinearity.

| **Cluster** | Base-Band | | In-Band | | | | 2nd Harmonics | | | 3rd Harmonics | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Mixing Terms** | DC | RF-LO | 2LO-RF | LO | RF | 2RF-LO | 2LO | LO+RF | 2RF | 2LO+RF | 3LO | 3RF | 2RF+LO |

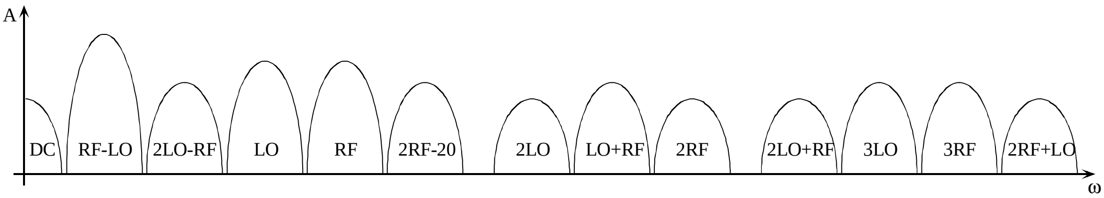



Fig.4 Output mapped spectrum. corresponding to the example shown on Tab.2.

Tab.2– Total number of frequency positions handled in the diamond/diamond truncation AFM.

| Order Parity | Number of real terms | Number of Zeros | Total Number |
|---|---|---|---|
| Even | $N_t \approx \frac{13O^2 - 10O}{8}$ | $N_z \approx \frac{2O^2 - 2O + 4}{4}$ | $N \approx \frac{O}{2}(N_t + N_z)$ |
| Odd | $N_t \approx \frac{13O^2 - 12O + 7}{8}$ | $N_z \approx \frac{2O^2 - 2O + 4}{4}$ | $N \approx \frac{O+1}{2}(N_t + N_z)$ |

## III. Comparison of the Two Proposed AFMs

Although by now it should be obvious that the diamond/box truncation AFM is more suitable for the simulation of quasi-linear mixers, while the diamond/diamond truncation AFM should be used for simulating mixers driven into saturation, the frontier between these two asymptotic behaviors is still unclear. Therefore, a remaining question dealing with the selection of the most appropriate technique for a particular case must be addressed.

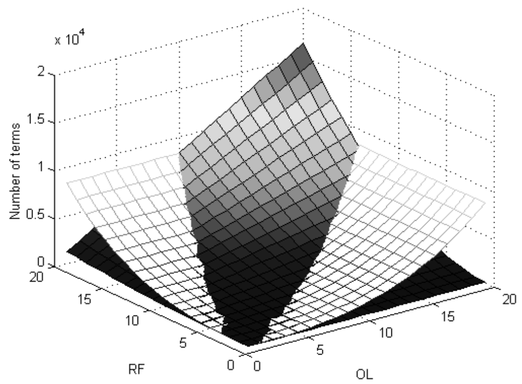



Fig.5 Relation between the size of the frequency index vector resulting from diamond/diamond truncation (filled), and diamond/diamond truncation (unfilled) AFMs.

For answering this question, the number of terms arising from the application of each of the discussed AFMs, as a function of the orders $O_{RF}$ and $O_{LO}$, is presented on Fig.5, for a very large number of input tones.

The *xx* axis of Fig. 5 is the LO order, the *yy* axis is the RF order, and the *zz* axis is the number of frequency positions required in the index vector.

Considering, for obvious reasons, that $O_{LO} \geq O_{RF}$ (right octant in Fig. 5), it is possible to conclude that diamond/diamond truncation AFM is only preferable in cases of similar $O_{LO}$ and $O_{RF}$. In any other situation, diamond/box truncation AFM is better. For example, for $O_{LO}$=9 and $O_{RF}$=3, $N_{diam/diam} \approx 743$ and $N_{box/diam} \approx 171$, but if $O_{LO}$=9 and $O_{RF}$=7, then $N_{diam/diam} \approx 743$ and $N_{box/diam} \approx 931$.

In summary, if a small-signal mixer is to be simulated - $O_{RF} \ll O_{LO}$ - then mixed diamond/box truncation scheme should be used, in order to minimize computational workload. If, on the other hand, the mixer stimulus is a large-signal RF excitation, $O_{RF} \approx O_{LO}$, then a completely diamond truncation scheme AFM is the correct option. This conclusion is quite surprising because, in this case, the resulting spectrum is not dense. The requirement of a periodic frequency index vector imposed the insertion of a certain number of zeros, and, even so, the total number of frequency index vector positions to be simulated remained smaller.

## IV. Real Mixer Illustrative Simulation

For illustrative purposes, an RF cold-FET mixer was simulated in its large-signal operation regimes. The frequency index vector used in the HB analysis, is based on the completely diamond truncation artificial frequency mapping. Fig. 6 presents a simplified diagram of the RF cold-FET mixer circuit.

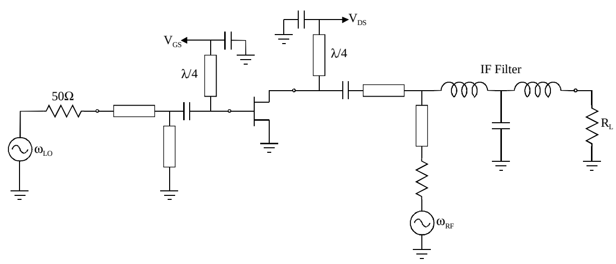
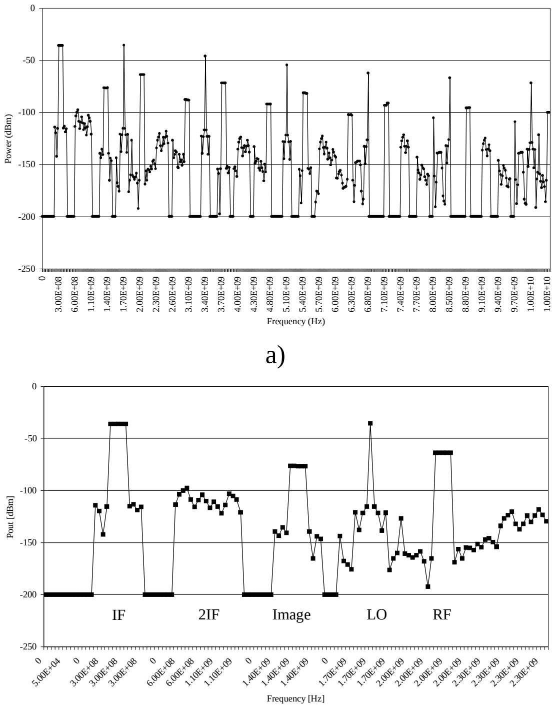



Fig.6 Simplified diagram of the cold-FET mixer circuit used in the simulation examples.

A LO of 1.7GHz and a 5-tone RF input signal near 2GHz were considered as the stimulus of an in-house AFM-HB simulator [3]. Fig. 7.a) and b) present the complete output signal spectrum, and its $0^{th}$ and $1^{st}$ order LO components, respectively.




a)

b)

Fig.7 Complete output signal, a); $0^{th}$ and $1^{st}$ order LO components b).

Fig. 7.a), shows the entire simulated spectrum. The values of –200dBm (approximately null power), are the adopted numeric representation of the zeros used to convert the mapped spectrum into a periodic one. The IF spectrum is the sought output frequency band, as depicted in Fig. 7.b). There, it is possible to see the fundamental output spectrum, and adjacent spectral regrowth components. In the same figure the other three most important bands of the mixer are also represented: RF input, LO and the image frequency. A precise control of the terminating impedances, and incoming signals of these four bands, plays a key role on mixer design and performance optimization.

## V. Conclusion

Two new artificial frequency mapping techniques specially conceived for the simulation of quasi-linear and saturated RF/microwave mixers subject to multi-tone excitations were proposed and discussed.

The complete diamond AFM technique was then validated by the HB analysis of a real cold-FET mixer.

With the aid of this new CAD capability it is now possible to efficiently analyze and design general small-signal or saturated frequency converter circuits.

## Acknowledgement

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